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Mass Distribution Model for Composite Airframes

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2. Executive Summary

Because of a demand for faster, more efficient aircraft, researchers are exploring the possibility of making wing structures lighter and more flexible. The design and control of a flexible wing aircraft is a more complicated challenge compared to traditional stiff wing aircraft. One of the larger challenges in this research area is predicting and controlling flutter, an aeroelastic phenomenon that can destroy and aircraft that reaches a velocity outside of its rated speed envelope.

In order to design and control flexible wing aircraft, it is necessary to create an accurate mass distribution model that is more sophisticated than what is required for a stiff wing aircraft. The rigid body assumption that the mass can be lumped in a single location in the aircraft no longer holds for flexible wing aircraft. Instead, the aircraft must be broken into a series of masses that can move relative to each other.

This report documents the development of two mass distribution methods for composite airframe shells and compares them using experimental density data. The first model was built in SolidWorks with surfaces, and those surfaces are broken into thousands of point masses in Excel. The second model was built in SolidWorks with solid bodies with assigned densities. It was determined that the user of these models would have more control over the solid body method, since they could use the SolidWorks graphical interface to extract the data that they need. Whereas, they would need to do more work to process the surface data in a program like Excel or Matlab. It was also found that the solid body method produced results within 1% of physical measurements on the composite airframe, and the surface body method was off by 18%.

It was recommended that the solid body method be used going forward, and further work needs to be done to model the rest of the aircraft's mass distribution. It is also recommended that once a complete mass distribution model is complete, the overall weight and the inertia be measured and compared to the model.

3. Glossary

CfAR	UVic Centre for Aerospace Research
OML	Outer mold line. Refers to the outer surface of an aircraft.
CAD	Computer Aided Design
STL	Stereo lithography file format. A type of CAD file
Layup	A process of building up a structure using layers of composite material
GUI	Graphical user interface

4. Introduction

Current research at CfAR is being done on the feasibility of flexible wing aircraft. These aircraft are theoretically lighter than traditional aircraft because they have less structural mass. Novel methods for the design, analysis, and control of flexible wing aircraft may be needed. This report focuses on the creation of a model in SolidWorks such that a composite airframe's mass distribution data can be easily exported based on the flexible-wing designer's preferred level of granularity.

5. Background

Traditional aircraft have been limited to a maximum safe airspeed due to the dangers of aeroelastic flutter. Flutter occurs when an airframe's structure (stiffness and mass) couple with aerodynamic forces at a critical airspeed, thereby causing uncontrolled oscillations that have historically caused the loss of many aircraft [1]. Traditional aircraft are limited to speeds that do not allow these uncontrolled oscillations. They are generally designed to have very stiff structures, because added stiffness can increase the critical airspeed that uncontrolled flutter will occur, and it can also decrease the oscillation amplitude [2] [3]. Added stiffness requires added structural mass, which decreases the efficiency of the aircraft. In summary, flutter limits the speed and efficiency of traditionally designed aircraft.

5.1. Composite Airframes

Within the limits imposed by aeroelastic flutter, aerospace engineers have used modern material science to reduce the weight of aircraft. This is particularly true for reducing structural mass, where composite materials have been able to either supplement or completely replace steel, aluminum, and other metal alloys. Figure 1 shows that composite materials can have a similar stiffness range to metals and alloys while having significantly lower mass density.

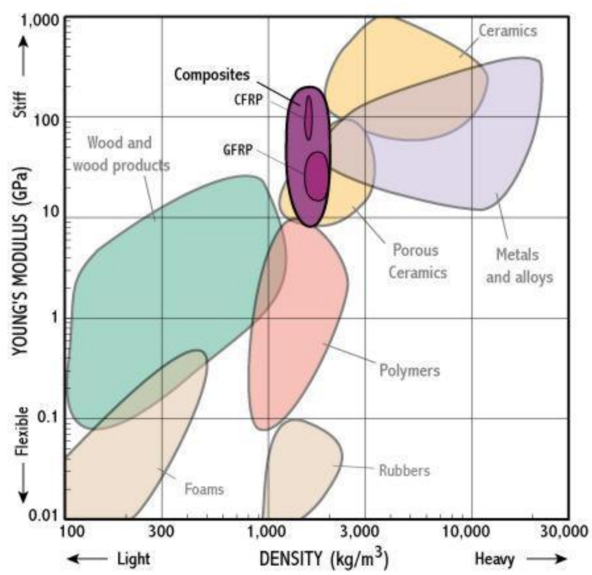

Figure 1: Young's Modulus (stiffness) vs Mass Density for common engineering materials [4].

5.2. Flexible Wing Flutter Control

Increases to airspeed and efficiency motivate research into active flutter control systems. Such research aims to allow light, flexible aircraft wings to replace the traditional stiff, heavy wings. Control systems – beyond the scope of this report – can be used to actuate controls surfaces and counteract flutter [3]. NASA's X56-A flexible wing aircraft, shown in Figure 2, is the most notable example of this being achieved.

Figure 2: NASA's X-56A Flexible Wing Aircraft [5]

5.3. Aircraft Inertia Measurement

The mass and inertia information must be known to the aircraft designer and control system engineer. These can be estimated ahead of time with hand calculations and design software such as SolidWorks. This is, in fact, the purpose of this report. However, these values should be verified experimentally before flight. The mass can be easily measured on scales and the center of mass calculated from that information with sum of forces and moments.

There are also methods for measuring moments of inertia. A common way to do this is with a bifilar pendulum [6]. This is done by hanging the aircraft by a set of long ropes, rotating the aircraft slightly from rest about the vertical axis, and releasing. The aircraft will oscillate in yaw, and the frequency of that oscillation can be used to calculate the moment of inertia about the vertical axis.

6. Problem

Stiff wing aircraft can, usually, be safely treated as a single rigid body moving through space. The designer needs to ensure the structure is strong enough for the required loading conditions, and they also must ensure that it is stiff enough that any deflections can be safely ignored in the aerodynamic analysis. The latter is also important for the control system engineer. These simplifying assumptions are no longer valid for the design and control of a flexible wing aircraft.

A single rigid body may be described by its mass m, center of mass **C**, and inertia tensor **I**, where

$$\mathbf{C} = \begin{bmatrix} c_x \\ c_y \\ c_z \end{bmatrix}$$

$$\mathbf{I} = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{xy} & I_{yy} & I_{yz} \\ I_{xz} & I_{yz} & I_{zz} \end{bmatrix}$$

It is common engineering practice to simplify a rigid body as a point mass with these characteristic values as it moves through space. A flexible wing aircraft must be modelled as multiple bodies that move relative to each other. Traditional inertia measurement methods (see Section 5.3) are not sufficient, as they only give a single set of information about the entire aircraft. Herein lies the problem: without a sufficient measurement technique, how do the designer and control system engineer model the aircraft accurately?

Problem statement: *There is currently no accurate way to know the mass distribution information sufficient for the design and control of a flexible wing aircraft.*

7. Objective

The objective of this analysis is to choose a mass distribution modelling method that is sufficient to design and control a flexible wing aircraft. Two methods are compared based on accuracy and practical usability.

8. Discussion

This analysis focuses on the mass modelling of composite airframes. Non-composite structure and components are not considered for this analysis. However, in most cases it is straight forward to calculate the mass and inertia information for non-composite components because they are often homogeneous in density. Mass and inertia information can be accurately modelled for these types of components with SolidWorks. The reason for focusing on the composite structures is that they are in many cases 'curvy' (formed to the aerodynamic OML), they have varying thickness, and they have varying density.

8.1. Composite Density Measurements

In order to create these mass models for composite structures, the density of the relevant materials needed to be known. A series of test coupons were made at Harwood Custom Composites in Sidney, BC. An image of the eight 4"x4" coupons is shown in Figure 3.

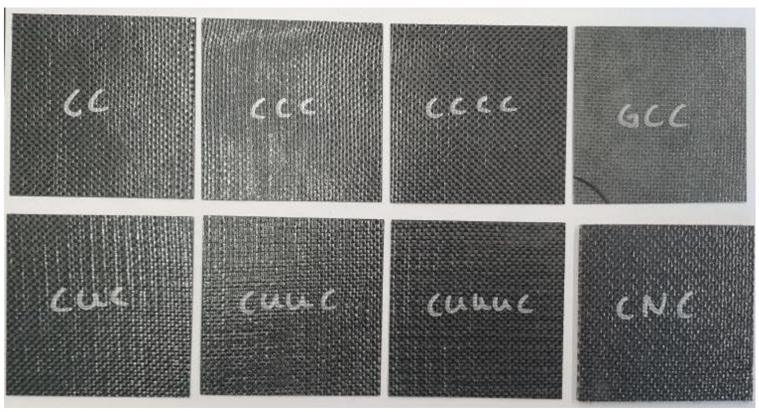

Figure 3: Composite test coupons

The letters on each sample show the layup order and material used in each coupon. The letters' material type is shown in Table 1.

Table 1: Letter Identifiers for Test Coupons

Letter Identifier	Material
C	Woven Carbon
U	Uni-Directional Carbon
G	Woven Glass
N	Nomex Honeycomb Core

Each coupon's thickness was measured with calipers in four locations and weighed on two different scales; the outer dimensions were also measured with calipers. The data are shown in Table 2.

Table 2: Sample coupon data.

Sample Name	W1 (g)	W2 (g)	T1 (in)	T2 (in)	T3 (in)	T4 (in)	D1 (in)	D2 (in)
CC	5.4	5.5	0.0195	0.0190	0.0210	0.0195	3.9990	3.9910
CCC	8.2	8.2	0.0260	0.0260	0.0260	0.0270	3.9925	3.9900
CCCC	10.9	10.9	0.0360	0.0365	0.0380	0.0355	4.0030	4.0015
CUC	7.7	7.7	0.0275	0.0270	0.0280	0.0270	4.0035	3.9920
CUUC	9.7	9.8	0.0355	0.0335	0.0330	0.0340	4.0000	3.9905
CUUUC	11.8	11.8	0.0400	0.0400	0.0410	0.0395	3.9765	4.0065
GCC	9.6	9.6	0.0270	0.0265	0.0270	0.0275	4.0120	4.0060
CNC	10.8	10.8	0.1465	0.1455	0.1465	0.1470	4.0000	4.0010

Table 3 shows data that was calculated from the raw data.

Table 3: Sample coupon calculated data.

Sample Name	W (g)	Tav (in)	Area (in2)	Volume (in3)	W/Area (g/in2)	W/Vol (g/in3)
CC	0.019750	15.9600	0.31521	0.34148	17.2901	0.019750
CCC	0.026250	15.9301	0.41816	0.51475	19.6095	0.026250
CCCC	0.036500	16.0180	0.58466	0.68048	18.6434	0.036500
CUC	0.027375	15.9820	0.43751	0.48179	17.5997	0.027375
CUUC	0.034000	15.9620	0.54271	0.61083	17.9655	0.034000
CUUUC	0.040125	15.9318	0.63927	0.74065	18.4587	0.040125
GCC	0.027000	16.0721	0.43395	0.59731	22.1226	0.027000
CNC	0.146375	16.0040	2.34259	0.67483	4.6103	0.146375

These data are used in the following sections to calculate the area density (g/in^2) and volume density (g/in^3).

8.2. Method 1: Surface Model

In order to create the area-based mass model, the OML was imported as a single surface body. It was then split into multiple surface bodies based on their layup schedule. Figure 4 shows a SolidWorks screenshot of two sections from the top part of a wing[1]. The red and blue areas are made with different composite materials, and therefore have different densities. They are modelled here as surface bodies with zero thickness, so SolidWorks cannot calculate their mass.

[1] For simplicity, only a small part of the wing is shown.

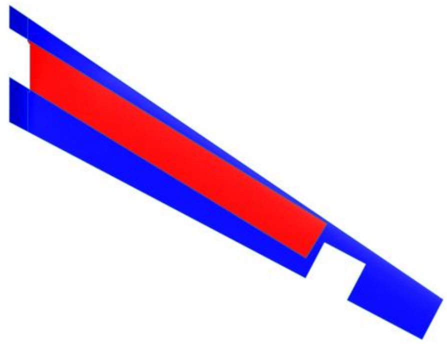

Figure 4: Plan view of the top surfaces of a wing.
The red and blue sections have different layup schedules.

Since SolidWorks cannot calculate the mass of these zero-thickness surface bodies, the geometry is exported and the mass is calculated in post. Each material type must be exported separately, so the blue surface is suppressed, leaving only the red as shown in Figure 5.

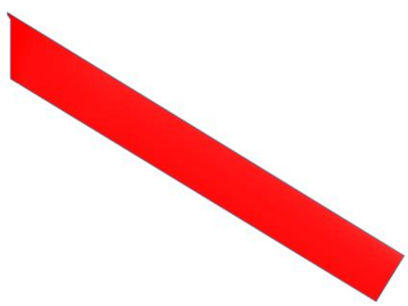

Figure 5: Red surface isolated.

Once isolated, the surface is exported as an STL, as shown in Figure 6. The grey lines represent the edges of the planar triangle elements.

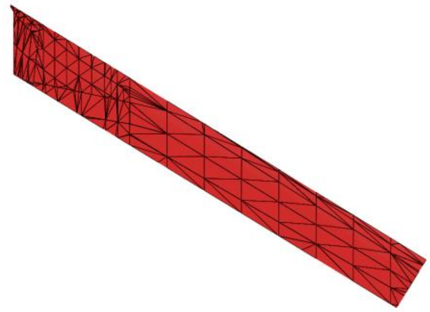

Figure 6: Red surface exported as an STL. Note the grey lines.

Once exported, the filename extension is changed from .stl to .txt. It can now be read in any text editor and will have the following form:

```
solid Red Wing Surface
   facet normal 7.586227e-02 8.734833e-01 -4.809072e-01
      outer loop
         vertex 5.803342e+02 2.113885e+01 -2.028639e+02
         vertex 5.763695e+02 2.305993e+01 -2.000000e+02
         vertex 5.973605e+02 2.123685e+01 -2.000000e+02
      endloop
   endfacet
   facet normal 6.616631e-02 9.031120e-01 -4.242768e-01
      outer loop
         vertex 5.803342e+02 2.113885e+01 -2.028639e+02
         vertex 5.973605e+02 2.123685e+01 -2.000000e+02
         vertex 5.864443e+02 1.841095e+01 -2.077176e+02
      endloop
   endfacet
..............................
..............................
   facet normal 7.551206e-02 9.841023e-01 -1.607500e-01
      outer loop
         vertex 6.012341e+02 1.144556e+01 -2.344880e+02
         vertex 5.981001e+02 1.429313e+01 -2.185275e+02
         vertex 6.033900e+02 1.297618e+01 -2.241049e+02
      endloop
   endfacet
   facet normal 9.932857e-04 9.892781e-01 -1.460407e-01
      outer loop
         vertex 6.012341e+02 1.144556e+01 -2.344880e+02
         vertex 6.033900e+02 1.297618e+01 -2.241049e+02
         vertex 6.113176e+02 1.161576e+01 -2.332665e+02
      endloop
   endfacet
endsolid
```

The .txt file displays the vertices of every triangle as a triplet of point coordinates based on the SolidWorks origin in mm; it is truncated above to only show the first and last two triangle elements. The next step is to open this file in excel as a space-delimited file and process the data. A simple VBA Macro was written to calculate the centroid and area of each mass element. The area was calculated using Heron's Formula [7] with the three vertices (see Appendix A) , and the centroid is simply the average of the three vertices.

The next step is to use the data from Section 8.1 to calculate the mass of each triangle element, based on its area. The three C, CC, and CCC data are used to calculate the mass per area for a layer of woven carbon. This is shown in Figure 7, where the slope of the line is the mass per area added for each woven carbon layer.

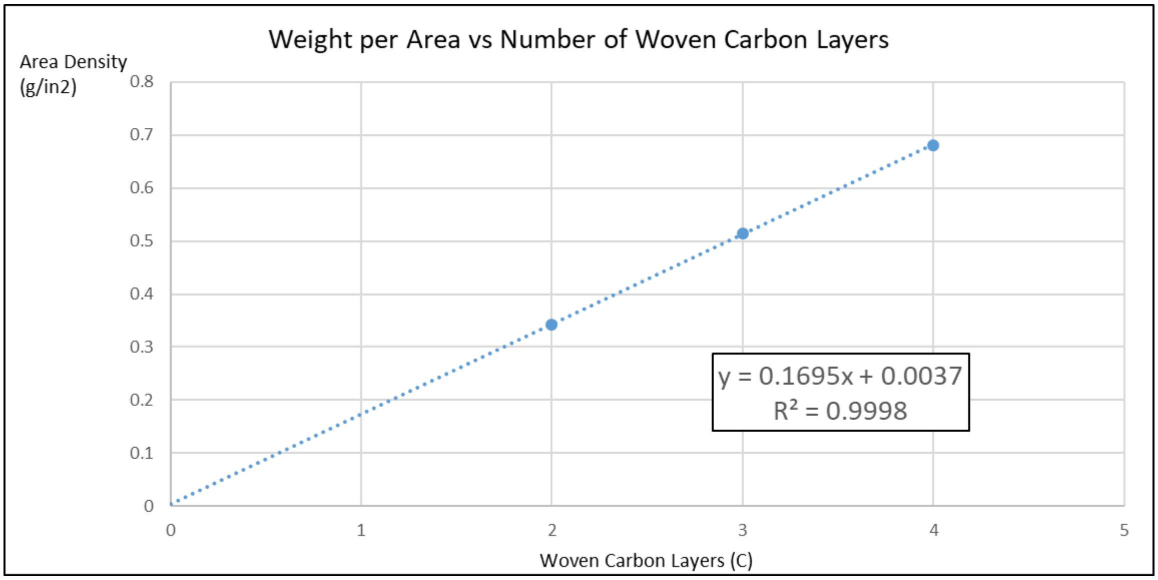

Figure 7: Woven carbon mass per area vs number of layers.

It can be seen in Figure 7 that a 1in^2 layer of woven carbon weighs 0.1695grams. Similar analysis was done for each type (C, U, G, N) and are summarized in Table 4

Table 4: Mass per area per layer for each material type.

Type	**Mass/area/layer (g/in^2/layer)**
Woven Carbon (C)	0.1695
Uni-Directional Carbon (U)	0.1327
Woven Glass (G)	0.255831
Nomex Honeycomb (N)	0.333353

With this information, an equation can be used to calculate the mass of each triangle element.

$$Mass = Area \times (0.1695C + 0.1327U + 0.25583G + 0.33335N) \quad [Equation\ 1]$$

Where C, U, G, and N are the number of layers of their respective materials. As an example, if the red surface body represents a composited structure with the sequence CCCUUNCCCCG, and some particular triangle element had an area of 0.8in^2, then the mass would be

$$Mass = 0.8\big(0.1695(7) + 0.1327(2) + 0.25583(2) + 0.33335(1)\big) = 1.383\text{grams}$$

After calculating the location and mass of each triangle element of the red surface, the process is repeated for the blue surface, and then every other surface in the airframe; this was all done in Excel. The result is a data set with many thousands of point masses representing the entire airframe.

8.3. Method 2: Solid Model

Like the first method, the aircraft OML is imported into SolidWorks as a surface body, and it is sliced into several different surfaces, each representing a different layup schedule. A close-up example of this is shown in Figure 8, where each colour represents a unique layup type. Each of these surfaces will therefore have a unique thickness and density.

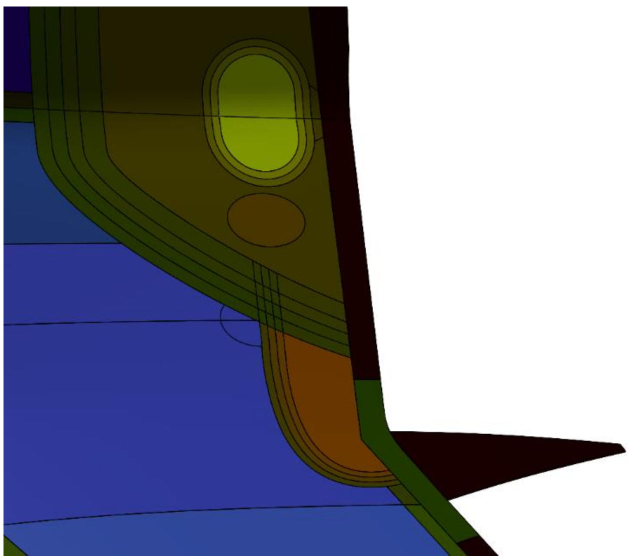

Figure 8: Close-up top view of a wing section.

The next step is to thicken each of these surfaces into a solid body using the ‘Thicken’ tool in SolidWorks. A series of thickened sections can be seen in Figure 9. Note that sections are thickened toward the inside of the airframe, so the outer surface is still smooth and accurate to the original OML.

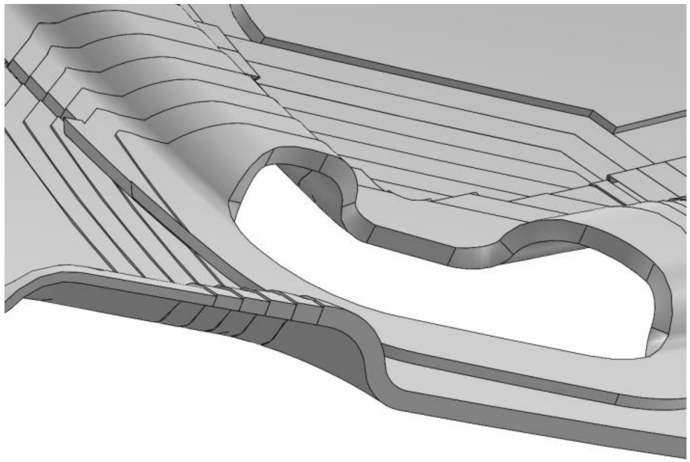

Figure 9: Thickened sections

Each thickened section is a unique solid body in the SolidWorks model. They are assigned thicknesses and densities based on the data from Section 8.1.

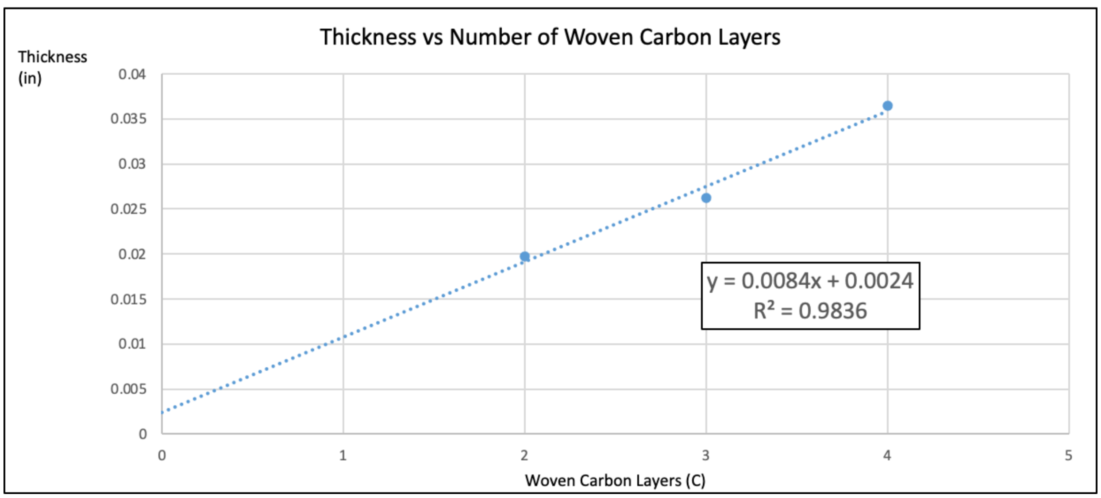

Figure 10: Thickness vs Woven Carbon Layers

The thickness of each woven carbon layer is given by the slope of the line in Figure 10, so each layer is 0.0084in thick. Similar analysis was done to the other samples, and the thickness per layer for each material type is shown in Table 5.

Table 5: Thickness per layer for each material type

Type	**Thickness/Layer (in/layer)**
Woven Carbon (C)	0.0084
Uni-Directional Carbon (U)	0.0068
Woven Glass (G)	0.00725
Nomex Honeycomb (N)	0.1266

The next step is to calculate the density of each material type. The density of the woven carbon (C) coupons is a simple calculation because the mass and dimensions are known. Referring to

Table 3, there are three experimental densities for woven carbon (C, CC, CCC), these averaging to 18.51g/in^2.

The measured density of the other coupons needs to have the woven carbon portion 'removed' from each sample to calculate the density of each material on its own. Figure 11 shows a sketch of a composite section with two material types[2] with densities ρ_1 and ρ_2, and thicknesses t_1 and t_2. The goal is to assign it an equivalent, average density ρ as if the entire composite section were homogeneous.

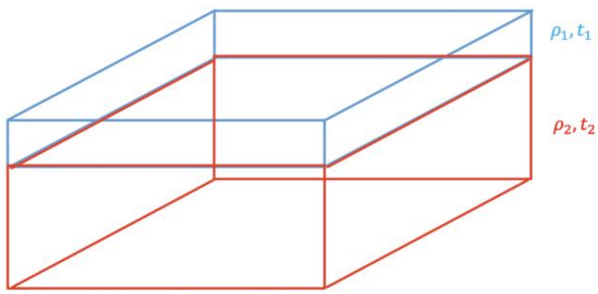

Figure 11: Composite example section

Since they are assumed to have to same area, the equivalent density is simply the average of the constituent layers' densities, weighted by thickness. The equivalent average density is therefore

$$\rho = \frac{\rho_1 t_1 + \rho_2 t_2}{t_1 + t_2} \quad [Equation\ 2]$$

This is generalizable for a section with any number of layers. By rearranging Equation 2, the other experimental densities can be found since the woven carbon density is known. For example, for the samples with both woven carbon (C) and uni-directional carbon (U), the density of the uni-directional carbon is found by

$$\rho_U = \frac{\rho_{sample}(t_U + t_C) - \rho_C t_C}{t_U}$$

This was repeated for woven glass (G) and Nomex Honeycomb (N). The resulting density for each of the four materials is shown in Table 6.

Table 6: Density of each material

Type	**Density (g/in^3)**
Woven Carbon (C)	18.51

[2] For example, it could be some layers of woven carbon and some layers of woven glass.

Uni-Directional Carbon (U)	18.97
Woven Glass (G)	35.62
Nomex Honeycomb (N)	2.64

The thickness of each thickened surface is determined by the values found in Table 5 and repeated in equation form here:

$$t = t_C + t_U + t_G + t_N$$

$$= 0.0084\text{C} + 0.0068\text{U} + 0.00725\text{G} + 0.1266\text{N} \;\; [Equation\ 3]$$

The density of each thickened surface is given by the average density of each material, weighted by thickness:

$$\rho = \frac{\rho_C t_C + \rho_U t_U + \rho_G t_G + \rho_N t_n}{t} \;\; [Equation\ 4]$$

Note that Equations 3 and 4 can be extended to include more materials with known thickness and density. For simplicity in this report, they were limited to the 4 materials tested in Section 8.1.

Once every surface was thickened and assigned a density, the mass model was complete. Extraction of useful mass and inertia data is left for the user (designer, control engineer, etc.). Since the model is a SolidWorks multibody part, it can be cut into any number of separate bodies (with any shape), and they will each have their own mass and inertia information.

8.4. Comparison of Methods 1 & 2

Both models have limited accuracy, in that they do not consider discrepancies between the manufactured airframe and the CAD model. Variation in epoxy/resin used during the layup process is not accounted for – such as in the transition between layup sections. Method 1 was done on Airframe 1, and Method 2 was done on Airframe 2. Their measured masses are compared to the model masses in Table 7.

Method 1 (Surface Model) consists of thousands of point masses, so the user may need to lump several local masses together depending on their needs. Also, local moments of inertia or products of inertia are not calculated for each triangle element (about their local center of mass) since they are assumed to be point masses. This is a good assumption only if the triangle elements are sufficiently small. Also, since the surfaces have zero thickness, their center of mass locations will be slightly shifted outward – this shift will be more pronounced for thicker sections. This method's total calculated mass was compared to measurements of the real airframe. **The modelled mass was 18% smaller than the physical measurement**. This was relatively unsurprising because this airframe was one of the smaller ones built by CfAR, and there was a large amount of glue holding the measured airframe together that was not accounted for in the model but could account for a larger portion of its mass. No mass measurement were done on the layup sections before they were glued together with multiple structural members. The measured mass had to be estimated by taking the overall measured mass and subtracting 0.375kg to account for items that had already been glued in.

Method 2 (Solid Model) requires the user to cut up the model using standard SolidWorks part editing features, 'Cut-Extrude' for example. The user may cut the model to any level of granularity they prefer. However, it may be labour intensive to achieve the granularity of the surface model. This model is expected to be more accurate than Method 1 because it takes into account the thickness of every layup. It also calculates the entire inertia matrix **I**, which Method 1 does not do. This method's overall mass result was compared to measurements of a real airframe (a larger version of the one used for Method 1). **The modelled mass was 0.98% smaller than the physical measurement**. This is an excellent result, indicating not only the validity of this method, but the consistency of the work done at Harwood Custom Composites. In fact, the accuracy of the weigh scale used would account for a 1% error.

Table 7: Model masses compared to measured masses.

Method	Airframe	Measured Mass (kg)	Model Mass (kg)	% Difference
Surface Model	Airframe 1	2.81	2.29	18.3
Solid Model	Airframe 2	11.55	11.44	0.98

9. Conclusion

Based on a need for faster, more efficient aircraft, research is being done on flexible wing aircraft. An important factor for the design and control of flexible wing aircraft is a detailed mass distribution model. This report focuses on a mass distribution model for composite airframes. The first step to building the model was the collection of density data for the relevant materials by weighing and measuring test coupons.

Once the data was collected, two methods for creating a mass distribution model were developed. Method 1 was done by exporting SolidWorks surface bodies as STL files, then parsing the STL file to compute the mass and location of every triangle element. Method 2 was done by thickening SolidWorks surface bodies and assigning them a density.

Although Method 1 may be better suited to certain applications, Method 2 is likely to be more user friendly and more accurate. Method 2 allows the user the break to model down into series of lumped masses however they prefer, using the SolidWorks GUI. Whereas, Method 1 requires the user to work mostly outside of a GUI, processing and lumping large amounts of mass data manually with software like Matlab or Excel. The difference in accuracy is also large, with Method 2 being only 0.98% different from measured results, and Method 1 being 18% off. However, the discrepancy for Method 1 is due at least in part to manufacturing method used for Aircraft 1.

10. Recommendations & Future Work

It is recommended that the mass model created using Method 2 (Solid Model) be used going forward. It is more user friendly and accurate than Method 1 (Surface Model). It has already been shown to agree with weight measurements on a real airframe. Since this model only contains the mass of the composite airframe, further modelling needs to be done for internal components, including any non-composite structural members. Once completed the entire model should be again verified using weigh scales. The inertia matrix should also be verified using the bifilar pendulum test described in Section 5.3. Another way to validate both methods would be to have more complex coupons made (such as the ‘CCCUUNCCCCG’ example from Section 8.2) and compare them against the models.

It is also recommended that CfAR have more test coupons made by their composite manufacturers. In the past, CfAR has focused on using test coupons to test strength and stiffness, so future tests should also include measuring and weighing the samples. The material testing documented in this report could be extended to more materials (including paint) and larger sample sizes. It may also be beneficial to analyze the effect of curved surfaces and stepped transition regions such as the one shown in Figure 9. It is likely that excess epoxy could add extra weight in these regions. All this information can be used to further refine mass models such as the ones shown in this report.

The relatively poor results for Method 1 may have been due to large amounts of glue in Airframe 1 that was not accounted for. It is recommended that a process be implemented where any glue containers be weighed before and after use, and the difference noted.

11. References

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[7] "Heron's formula," Wikipedia, [Online]. Available: https://en.wikipedia.org/wiki/Heron%27s_formula. [Accessed 24 August 2023].

Appendix A

Heron's Formula [7] takes the three sides lengths of any triangle to calculate the area. Since the three vertices are known, the side lengths are calculated as the distance between them. The area is then given by Heron's Formula:

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

Where s is the semi-perimeter

$$s = \frac{1}{2}(a + b + c)$$